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The Determination of the Specific Heat of Superheated Steam by Throttling and other Experiments.

By A. H. PEAKE, M.A., A.I.E.E.

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In October, 1898, the author commenced experiments, having for their object the determination of the specific heat of superheated steam. At first an attempt was made to obtain this end by measuring the rise in temperature produced in a known quantity of steam by supplying a definite amount of heat in the form of electrical energy, but the experimental difficulties experienced in satisfactorily preventing radiation, in maintaining the rate of flow of steam uniform and in securing a steam supply sufficiently homogeneous and constant as to temperature, proved so great that the attempt on these lines was given up for a time, but returned to later.

Then another method was adopted, that of allowing dry saturated steam to expand without doing external work, and observing the resulting change in temperature. This method had been used in preliminary experiments on this subject by Professor Ewing and Mr. Dunkerley, who found that the specific heat of superheated steam at atmospheric pressure, as deduced by this method from Regnault's values of the " total heat," was not a constant, as had been previously supposed, but increased with temperature.*

While the writer's experiments were in progress, an account of an almost identical research, carried out by Mr. J. H. Grindley, was published in the 'Philosophical Transactions of the Royal Society,' A, vol. 194, pp. 1 to 36. This investigation covered so completely the ground which the author had intended to cover, and the results agreed so well with those obtained up to that time, as to discourage at first any further pursuit of the subject.

However, after careful examination of Mr. Grindley's paper and comparison of results, there appeared to be a somewhat serious source of error common to both his experiments and the author's, and it was decided to proceed with the investigation with the object of obtaining fresh data.

The results finally arrived at appear to amply justify this decision, the source of error, which will be mentioned presently, being gradually reduced and at last effectually eliminated. When this was done the conclusions,

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* Note read at the British Association, Toronto, 1897.

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while bearing out Mr. Grindley's work in several particulars, differed from it in at least one important feature.

It is well known that if dry saturated steam be allowed to expand without doing external work, and without losing heat, it becomes superheated. The direct object of the experiments was to determine the law connecting the temperature and pressure of superheated steam during this free expansion; then from this law and Regnault's tables of the total heat of saturated steam, the degree of superheat, and the amount of total heat corresponding to this superheat, can be obtained at any particular pressure lower than the initial pressure of the steam, and hence the mean specific heat over that range of superheat, and also the manner in which it varies—if the law be known for various initial pressures—may be calculated. This is, of course, assuming that Regnault's tables of saturated steam are accurate.

Until the throttling experiments were concluded, no doubt was entertained on this point, but it was then seen that a small deviation from the straight-line law laid down by Regnault, connecting total heat and temperature of saturated steam, would have a considerable effect on the calculated specific heat of superheated steam, and the author is now convinced that Regnault's tables are not sufficiently accurate to enable the specific heat to be obtained by this method with any degree of accuracy. In the meantime, however, as Regnault's tables are the only ones available for this purpose, they have been used in the calculations connected with the wire-drawing experiments, so that the final conclusions from these experiments respecting the value of the specific heat of superheated steam must await the correction of these tables.

Let p_1, t_1, H_1, v_1, and u_1 represent respectively the pressure, temperature, total heat, specific volume, and velocity of dry saturated steam before expansion.

Let p_2, t_2', v_2, and u_2 represent respectively the pressure, temperature, specific volume, and velocity of the steam after being superheated by free expansion.

Also let t_2 and H_2 represent the temperature and total heat of dry saturated steam at pressure p_2; let s be the volume of unit mass of water, let J stand for Joule's mechanical equivalent of heat, and let K_p be the mean specific heat of superheated steam at constant pressure p_2 between temperatures t_2 and t_2'.

Then the total energy of unit quantity of steam before expansion is

$$H_1 - \frac{p_1(v_1 - s)}{J} + \frac{u_1^2}{2gJ}.$$

The total energy after being superheated by expansion is

$$H_2 - \frac{p_2(v_2 - s)}{J} + \frac{u_2^2}{2gJ} + K_p(t_2' - t_2),$$

and the heat equivalent of the work done on the steam during the free expansion is $\frac{p_1 v_1 - p_2 v_2}{J}$, for $p_1 v_1$ is the work done on the steam entering the orifice by the boiler steam behind it, and $p_2 v_2$ is the work done by the steam coming from the orifice on the exhaust steam before it.

Now the final energy is equal to the initial energy together with the added energy, that is

$$H_1 + \frac{(p_1 - p_2)s}{J} + \frac{u_1^2 - u_2^2}{2gJ} = H_2 + K_p(t_2' - t_2),$$

so that the total heat is the same both before and after free expansion except for the terms $\frac{(p_1 - p_2)s}{J} + \frac{u_1^2 - u_2^2}{2gJ}$. Both of these quantities are small, and enter as slight correction into the results.

It is, of course, important that the saturated steam before expansion should be quite dry. Failure in this respect was the chief cause of difficulty experienced in the earlier experiments, but when this condition was at last obtained, the experimental results became at once much more simple and satisfactory.

Now in Mr. Grindley's experiments this condition of perfect dryness before expansion was never secured, for after the steam had been allowed to dry in a separator, it was conducted to the orifice disc where the throttling took place along a $\frac{3}{4}$-inch steam pipe about 1 foot in length, no mention is made of lagging round this pipe, and even if it had been well lagged this would not have entirely prevented radiation, consequently moisture must have been reproduced, the velocity of the steam in the pipe was most probably sufficient to prevent any separating action, and the moisture was carried along.

This defect in the design of his apparatus led him to the erroneous conclusion that what is ordinarily taken to be dry saturated steam, is not really quite dry, but contains a definite amount of moisture, the amount depending upon the temperature.

The principal piece of apparatus used in the author's experiments was virtually a form of throttling calorimeter, specially designed, and gradually improved to avoid, as far as possible, errors due to conduction and radiation, and to reduce corrections dependent upon the kinetic energy of the steam.

After a series of small improvements, the apparatus finally assumed the form shown in part section in fig. 1.

FIG. 1.

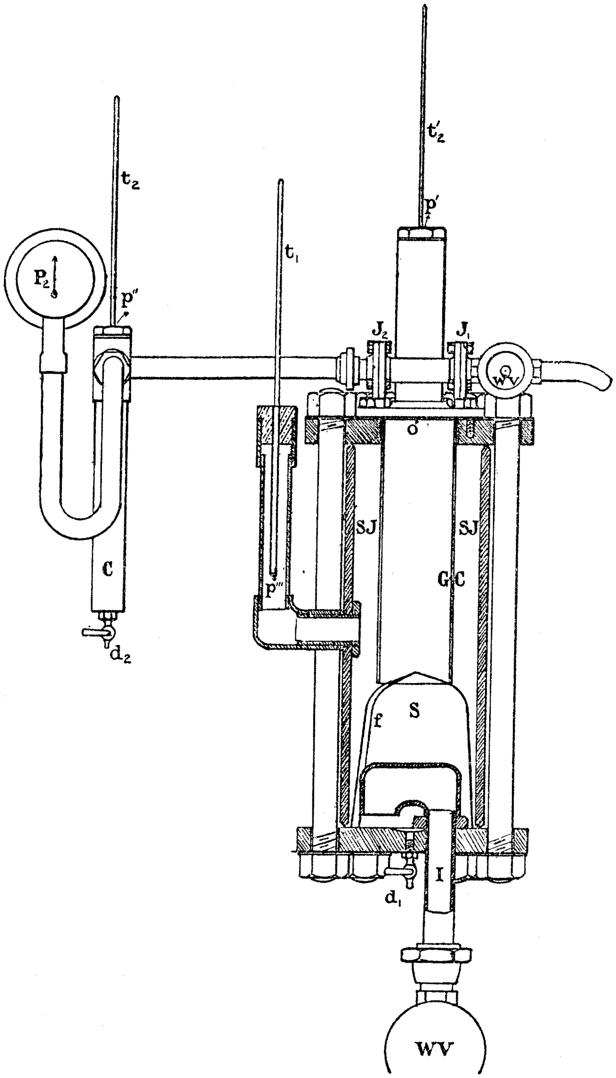
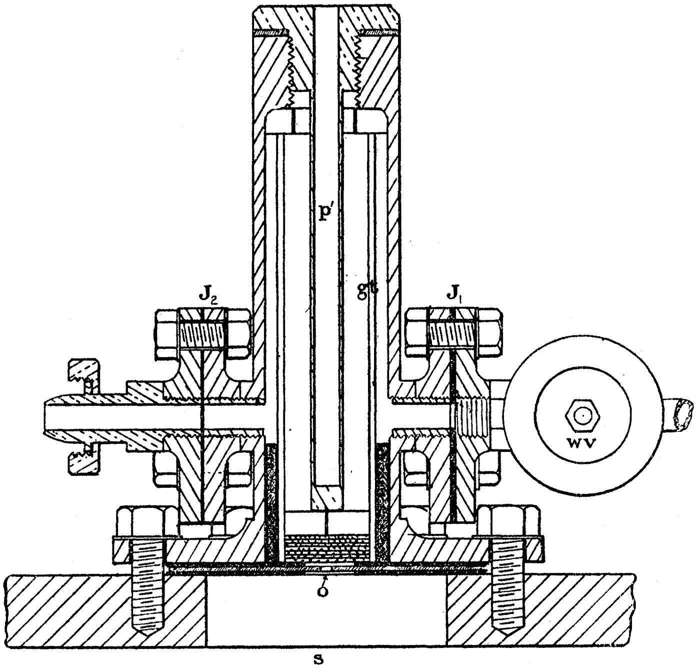


Fig. 2 gives, on a larger scale, a section view of the throttling calorimeter alone.

The steam was taken direct from the crown of a locomotive boiler—not shown in the figure—through a wheel-valve, WV, to a separator, S; it was then allowed to pass through a small orifice, O, in a mica disc, thus being wire-drawn, or throttled, and consequently superheated. This disc was held down on the top of the separator by a flange and studs, the steam joint being made by placing an asbestos washer on each side of the mica disc; these washers, which only needed a small inside diameter, also materially assisted the mica in the prevention of conduction of heat from the high-pressure steam to the cooler wire-drawn steam.

The now superheated steam coming from the orifice, immediately entered a glass tube, *gt*, and was caused to pass through several layers of fine copper gauze, whose object it was to destroy all eddies in the steam, and convert their kinetic energy into heat energy.

FIG. 2.



Directly after leaving the gauze, the steam came into contact with a thermometer pocket, *p'*, in which was a thermometer for taking the temperature, t_2', the pocket being concentric with, and inside of, the glass tube; after traversing the length of this tube, the steam passed down an annular space outside it—thus forming, so to speak, its own jacket—and was then allowed to escape into the air through a small wheel-valve, *wv*, which served to regulate the pressure of the superheated steam.

Since the temperature of this valve might be much lower than that of the steam entering it, on account of the throttling which was generally necessary there, the valve was connected to the apparatus by a heat-insulating joint, J. This joint was made by bolting together two flanges with asbestos packing between, asbestos washers also being placed under the bolt heads and nuts.

The whole of this part of the apparatus from the separator was thoroughly well lagged with felt; this lagging is not shown in the figures.

The pressure reading of the superheated steam was obtained during the more recent experiments in the following manner:—A passage from the side of the apparatus opposite the exhaust valve, *wv*, led through a heat-insulating joint, J_2, similar to the one just described, along an unlagged iron tube about 20 inches in length, to a small chamber, C, containing another thermometer pocket, p'', in this chamber the steam was allowed to become saturated—a process which never presented any difficulty—and the water formed was drained out by a cock, d_2, at the bottom, a small quantity of steam was allowed to blow through this drain-cock with the water, to ensure that water was not collecting about the thermometer pocket; the temperature, t_2, was taken there, and the pressure obtained from Regnault's tables. The flow of steam along the passage was so slight that no appreciable drop in pressure could be attributed to it.

A pressure-gauge, p_2, was also fitted; this was not relied on for the true reading, but it was a distinct convenience in working.

Several materials were tried for the orifice discs: glass, boxwood, earthenware, and mica; of these, mica was found to be the most satisfactory, the glass discs invariably cracked, the orifices in the wooden discs closed up, and the discs warped and split; earthenware discs were satisfactory in use, but were not so easily made as mica ones; consequently mica was the material adhered to after the preliminary experiments.

Three sizes of orifice were used, their diameters being respectively 3/64, 1/16, and 3/32 of an inch. The smallest size was used for boiler pressures of 150 lbs. per square inch and upwards, except when the pressure of the throttled steam was required to approach so near to the boiler pressure that the flow became excessively feeble, and a radiation error became apparent, in which case the next size was substituted. For initial pressures between 70 and 110 lbs. per square inch inclusive, the 1/16 inch orifice was used, and below 70 lbs. pressure, the largest one, 3/32 inch was used.

No great importance is attached to this, but these sizes were found to be best adapted to those pressures in this particular apparatus. If the orifices were much reduced sufficient steam did not pass to make the radiation loss inappreciable, if they were much increased the blast of steam was inconveniently large, and the gauze might be insufficient in quantity to destroy the eddies.

The separator was constructed out of a piece of 4-inch steam pipe, 12 inches long, the ends were cast-iron flanges, and were bolted together with six $\frac{3}{4}$-inch bolts; the effective diameter of the separator was, however, considerably reduced by the insertion concentrically of a glass cylinder, GC, a lamp chimney about 8 inches in length and $2\frac{1}{4}$ inches in diameter, which was supported upon a wire frame, *f*, in such a manner that its upper edge was in the same plane as the upper face of the top flange, the annular space round the glass cylinder thus becoming a steam jacket, SJ.

The steam inlet pipe, I, projected through the lower flange, and the direction of the entering steam was changed by means of two elbows, so that it blew against the flange. In this lower flange, also, was fixed the separator drain-cock, d_1.

The pressure reading of the steam in the separator was obtained by means of a temperature measurement and Regnault's tables. For this purpose a pipe projecting from the side of the separator contained a third thermometer pocket, p''', to the outside of which the steam had free access, and the pocket contained in turn the thermometer for obtaining temperature, t_1; any steam condensing in this pipe drained back into the separator.

Although the pressure in the separator was the same as that in the boiler under ordinary working conditions, yet it was preferred to obtain the pressure from the temperature readings, just as in the similar case previously mentioned, rather than trust to the boiler gauge readings, this method being found to give more accurate results, although the gauges were good ones of their kind, and had been carefully calibrated.

The thermometers used were mercury ones, graduated in divisions of half a degree Centigrade; they had been carefully calibrated previously by indirect comparison in hot oil well stirred with a standard platinum resistance thermometer, originally standardised by the author, and corrections necessary to give the temperature on the air scale were thus obtained to 0·1° C. over the required working range. They were also calibrated in position by blowing saturated steam through the apparatus, the orifice disc first having been removed; the pockets had been made equal in length, and were kept filled with oil just as during ordinary experiments. Although this was a check on the accuracy of the pressure gauges rather than the thermometers, yet the differences between the readings of the various thermometers agreed with those formerly obtained.

It is evident from consideration of the character of the results that a slight difference between the apparent temperatures and the true temperatures on the air scale would have no appreciable effect on the results; it is accuracy of the scale that is of vital importance, hence no correction need be applied for the effect of the thermometer pockets, since, as has just been shown, if the readings were affected at all they were affected equally.

The temperature readings taken during an experiment were converted into the Fahrenheit scale, as the English system of units was used throughout.

The method of conducting the experiments was as follows:—The boiler pressure was brought to some predetermined value, and maintained constant throughout a complete experiment by continual readings of the temperature of the boiler steam, the slightest observable tendency to change being checked by slight movements of the fire-door or dampers. Steam was then allowed to blow through the apparatus, the separator inlet valve being maintained full open during each experiment, the drain-cocks were opened by a suitable amount, and a few minutes allowed to elapse for the thermometer readings to become settled; as they approached this stage, and for a few minutes afterwards, readings of all three thermometers were taken at intervals of a minute or two. If the temperatures were subject to slight fluctuations, which was often the case, a larger number of readings was taken and the average obtained.

The pressure of the wire-drawn steam was raised step by step by slightly closing the exhaust valve, and the process of taking temperature readings was repeated at each step; by plotting the results with pressures as abscissæ and temperatures of the superheated steam as ordinates, a "cooling curve" could be drawn over which the total heat remained constant except for the very slight corrections before mentioned.

By repeating the experiments with various boiler pressures, a series of "cooling curves" or "constant total heat curves" graphically representing the law sought, was obtained.

Whenever commencing an experiment, if the apparatus was at all wet, it was necessary to allow steam to blow through it for a long time, 10 or 15 minutes, to completely dry that part which was to contain only superheated steam, for it was found that even when the superheat was as much as 50° C., a considerable time was necessary for this purpose, as was evinced by the slow rise in the thermometer reading. It was also necessary to be careful not to raise the pressure of the superheated steam too rapidly or by too great steps, especially when the amount of throttling was not very great, as the heat capacity of the apparatus might be sufficient to cause condensation to take place, and the progress of the experiment be considerably delayed in consequence.

The steam was obtained from locomotive boilers; in the earlier experiments one working up to a pressure of 80 lbs. per square inch was used, but later the apparatus was connected to the crown of a somewhat larger boiler—rated at 10 horse-power—working up to 200 lbs. per square inch, with which, owing to its capacity, it was much easier to maintain the steam pressure constant.

The boilers were sometimes filled with soft water, sometimes with hard water, either by means of an injector or a pump, but no difference in the results was ever traced to these changes. On getting up steam, the cocks of the water level gauges were left open for some time in order to get rid of air from the steam space. It was impossible to maintain the boiler pressure constant if steam was being taken from the boiler in any quantity for any other purpose, or when the boiler was being filled, so that experiments were only carried out when steam was not wanted elsewhere, and between boiler fillings. One filling often sufficed for two complete experiments.

With a short well lagged connecting stem between the boiler crown and the apparatus—about 9 inches of $\frac{3}{4}$-inch steam pipe and valve—the results, which agreed fairly well with those published by Mr. Grindley, were obtained, though the fall of temperature before the gaseous condition was reached, was not quite so great. This was before the separator was constructed.

The following brief account of alterations made at various times gives some idea of how the apparatus was gradually improved, and how the "cooling curves" or "constant total heat curves" obtained were thereby raised in greater or less steps until no part lay on the saturation curve, *i.e.*, the curve connecting temperature and pressure of saturated steam.

The length of connecting steam pipe on the low-pressure boiler was reduced until the lower flange of the apparatus—which was afterwards replaced by the separator and is, therefore, not shown in the figures—was as close as possible to the boiler crown, the valves between the apparatus and the boiler being removed. This was, of course, inconvenient, since no change could be made in the apparatus except when the boiler was cold, but the results thus obtained were only lacking in that they covered so small a range.

On the high-pressure boiler it was impracticable so to shorten the connecting pipe; it was, however, made as short as possible, but it was then found that the removal of the steam valve allowed steam to pass wetter than before, showing that the valve must have been acting to some extent as a baffle, causing separation of moisture.

Until these changes were tried it was by no means realised how much the results might be vitiated by quite a short length of well lagged connecting stem.

This fact explained to a large extent the cause of the difficulties met with earlier, and has also an important bearing on the use of throttling calorimeters generally, since in most cases wetness must be produced from this cause in the instruments themselves, before throttling takes place.

The separator was then constructed and placed on the high-pressure boiler, and the calorimeter placed directly on its crown. Up to this time the steam pressure was read from a test gauge fitted on the boiler, but afterwards it was derived from a temperature reading as previously described.

The glass cylinder inside the separator was also found to be an improvement; this was most probably due to the fact that the separator being of massive construction and unlagged, condensed a large amount of steam, and that portion of the water thus formed on the upper flange rained into the drying steam, unless prevented by the glass cylinder.

The wire gauze just above the orifice was put in the apparatus on first constructing it, but having on one occasion been accidentally omitted, it was found that its absence might cause under the most unfavourable conditions as much as 7° C. fall in temperature as measured by the thermometer, although the higher parts of the thermometer pocket were at a much higher temperature.

This was, no doubt, accounted for by the fact that the full strength of the steam jet from the orifice impinged directly on the base of the pocket, but a little further on the velocity had naturally died out to a great extent.

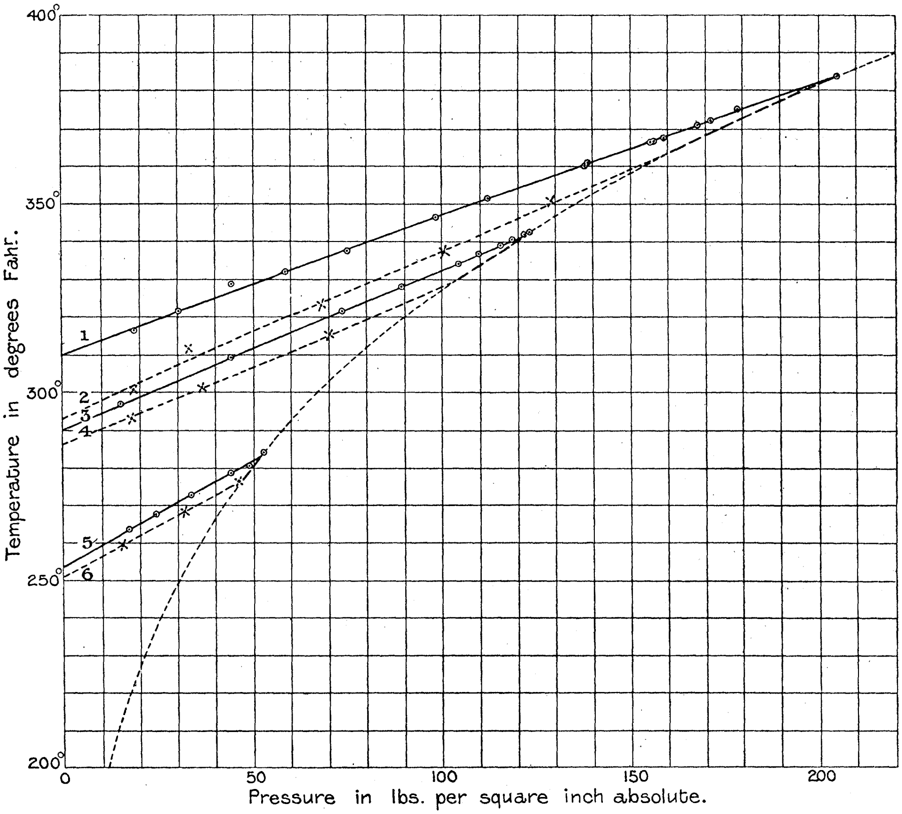
The heat-insulating steam joints, made by means of asbestos washers, which were inserted between that part of the apparatus containing the superheated steam and the colder leading away pipes, formed the last improvement made in the construction of the apparatus.

Fig. 3 gives cooling curves obtained at various stages during the experiments. Curves 1 and 2 show the sum of the errors caused by absence of gauze and radiation loss from a short length of well lagged connecting stem —about 5 inches; 1 was obtained using the separator and gauze, 2 was obtained before the separator was added, and without gauze in the apparatus; in each case the boiler pressure was 190 lbs. per square inch.

Curves 3 and 4 were both obtained with boiler pressure 109; Curves 5 and 6 both with boiler pressure 40 lbs. per square inch. 3 and 5 were obtained with the throttling calorimeter on the crown of the separator, whereas 4 and 6 were obtained without the separator, and show the vitiating effect due entirely to having a connecting steam pipe between the dry steam supply and the orifice, although this pipe was short and well lagged.

The apparatus having now been perfected as far as possible, a complete

Fig. 3.



new series of experiments was carried out, the corrected results of which are given in Table I. The cooling curves obtained by plotting these results are shown in fig. 4.

Before proceeding to the discussion of the curves it will be well to notice the probable and possible corrections. Experiments were made to furnish the necessary data for the calculation of the correction due to the energy of motion of the steam. The quantity of steam passing per minute under the various conditions existing during the experiments was determined by condensing all the steam passed during two or three minutes, and weighing the water thus formed. This quantity did not diminish appreciably until the pressure of the wire-drawn steam was about half that of the boiler steam, but as the pressure of the wire-drawn steam was still further increased, the quantity steadily diminished.

The maximum flow was 0·35 lb. per minute. This occurred with each of the three orifice discs under the maximum boiler pressure to which each was subjected, when the low-pressure side of the disc was atmospheric.

The area of the steam passage by the thermometer pocket was 0·32 square inch.

Assuming the maximum specific volume of steam met with in the course of the experiments—namely at 320° F. and at atmospheric pressure—to be 30, the heat-equivalent of the kinetic energy of the steam, at the point where the temperature of the throttled steam was taken, never exceeded 0·15 heat unit under the most unfavourable conditions experienced, if the eddies in the steam were overcome in a reasonably satisfactory manner by the use of the gauze, and this correction always falls off rapidly as the pressure of the wire-drawn steam is increased, since the specific volume is thereby diminished.

The kinetic energy of the steam on the high-pressure side of the disc is of a much inferior order and may be totally neglected.

Conduction and radiation were combated by the special design of the apparatus, and the sum of all possible errors, due to causes just enumerated, was shown to be constant and therefore most probably inappreciable, by the fact that similar curves were obtained by the use of orifices of different sizes, which of course allowed different quantities of steam to pass, thus changing the relative importance of the various errors. For examples of this see curves A and B, fig. 4. In each of these, different parts of the curves were obtained by different sized orifices (see table), but the overlapping parts coincide.

The change of total heat during free expansion due to the neglecting of the energy existing in the water at 32° F. in the estimation of the total heat, though slight, is perfectly definite, and corrections have been made accordingly before proceeding to calculation of the specific heat. Since this correction $s(p_1 - p_2)/J$ is to be expressed in British thermal units, $s(p_1 - p_2)$ must of course be in foot-lbs., therefore p_1 and p_2 must be the pressures in lbs. per square foot, and s must be 0·016, the volume of 1 lb. of water in cubic feet.

On examination of the set of curves in fig. 4 it is seen that they are practically straight lines, not quite parallel, but becoming slightly more horizontal with higher initial pressures.

However, it appears that if steam ever exists as a perfect gas, it is only at temperatures completely outside the range of these experiments, for the constant total heat curves would then be horizontal.

Although no pressures below that of the atmosphere were ever used in the experiments, yet the curves have been produced to the line of zero pressure, as there is no reason to suppose that the character of the curves suddenly changes in this short length, in fact it was proved by Mr. Grindley's

Table I.

Curve.	Pressure on boiler gauge.	Temperatures of steam.			Pressure from Regnault's tables.	Diameter of orifice in inches.
		Before expansion. t_1.	After expansion. t_2'.	Re-saturated. t_2.	p_2.	
A	191	383·5°	321·4°	250·9°	30·3	3/64
			316·6	224·4	18·7	
			328·8	272·8	43·9	
			332·1	290·8	58·4	
			337·8	307·4	75·0	
			347·0	326·5	98·5	
			351·3	335·8	111·9	
			360·5	351·7	137·9	
			367·0	361·8	156·8	
A	191	383·5	315·5	214·9	15·6	3/64
			321·6	248·0	28·8	
			327·2	273·0	44·0	
			332·8	292·1	59·6	
			338·3	306·3	73·8	
			349·3	330·6	104·2	
A	190	383·5	361·2	352·8	138·3	1/16
			366·8	361·0	155·3	
			367·9	362·8	159·0	
			371·3	367·3	167·9	
			372·4	369·1	171·7	
			375·1	372·4	178·8	
B	150	365·0	308·5	220·6	17·4	3/64
			313·7	245·3	27·5	
			318·9	270·1	42·0	
			326·1	294·1	61·5	
			332·6	209·9	77·8	
			338·4	323·1	94·0	
			343·8	333·5	108·4	
			350·8	345·2	126·7	
B	150	365·0	333·6	311·2	79·3	3/64
			346·5	337·2	113·8	
			350·3	343·4	123·8	
			353·8	349·4	133·9	
			355·1	351·3	137·2	
B	150	365·0	350·3	343·9	124·6	1/16
			351·9	346·5	128·9	
			353·7	349·3	133·7	
			354·9	351·3	137·2	
			357·4	354·9	143·7	
			363·7	363·7	159·3	
C	109	342·8	297·0	212·0	14·7	1/16
			321·8	306·1	73·6	
			328·1	318·4	89·2	
			334·5	330·4	104·2	
			337·1	334·2	109·5	
			339·3	338·0	115·2	
			340·9	340·0	118·3	
			309·0	272·8	43·9	
			342·1	342·1	121·7	

Table I—*continued.*

Curve.	Pressure on boiler gauge.	Temperatures of steam.			Pressure from Regnault's tables. p_2.	Diameter of orifice in inches.
		Before expansion. t_1.	After expansion. t_2'.	Re-saturated. t_2.		
D	71	315°·1	281°·9	215°·4	15·7	1/16
			285·9	334·1	22·4	
			291·5	257·0	33·7	
			292·1	258·3	34·5	
			296·4	271·2	42·8	
			301·6	286·0	54·2	
			305·8	296·2	63·5	
			310·4	306·5	74·0	
			312·8	311·4	79·6	
E	40	284·1	263·2	218·1	16·6	3/32
			267·4	237·9	24·1	
			272·0	255·4	32·8	
			278·7	272·9	44·0	
			280·2	279·5	48·9	
			284·0	284·0	52·6	
F	20	256·5	244·6	214·9	15·6	3/32
			250·5	239·7	24·9	
			253·5	249·3	29·5	
			256·6	256·5	33·4	

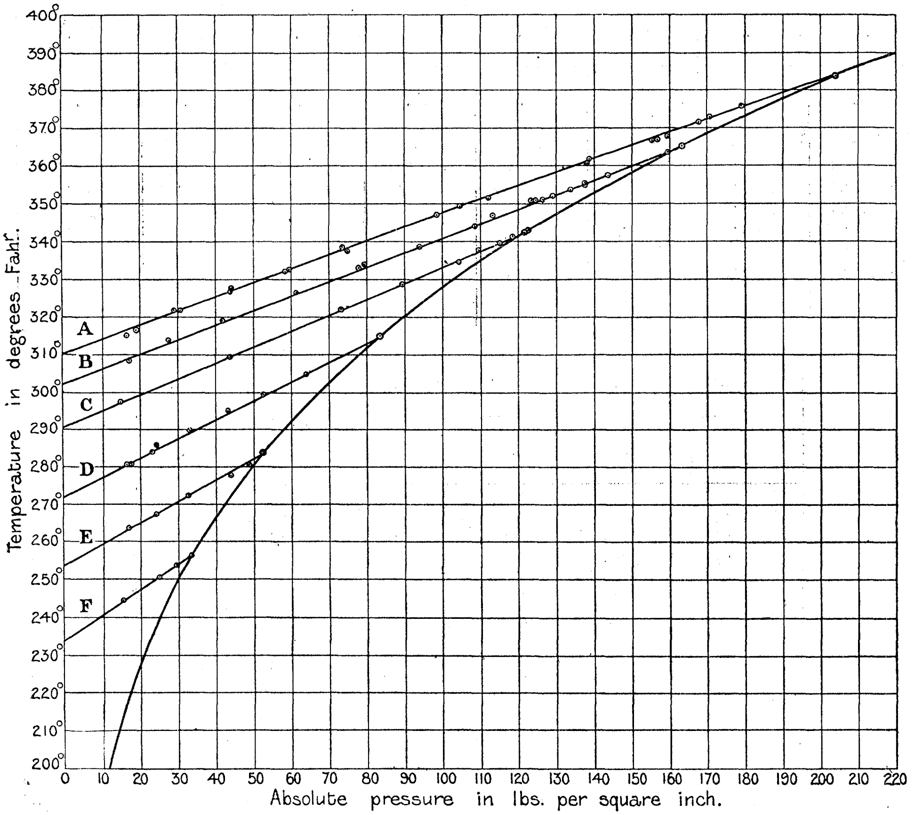
experiments that the character does not so change when the steam is throttled down to a pressure of 2 or 3 lbs. per square inch absolute.

The maximum boiler pressure available was, as previously stated, 200 lbs. per square inch; however, if a pressure of 336 lbs. absolute had been available, which is the maximum value given in Regnault's tables, and is therefore the limiting pressure by methods based on his figures, the upper limit of fig. 4 would only have been altered by a very small amount, since the additional total heat of saturated steam corresponding to this increase of pressure, is only 12·5 units.

From fig. 4, Table II has been constructed, showing the connection between total heat according to Regnault—corrected by the slight change in the total heat that takes place during free expansion as explained earlier—and temperature at constant pressure for various pressures, and the results contained in this table are represented graphically in fig. 5.

The slope of the curves in fig. 5 gives, then, the specific heat of steam at constant pressure, according to the author's experiments, based on Regnault's tables of the total heat of saturated steam. It will be noticed that where two or more curves cross the same ordinate, the slope is about the same for

Fig. 4.



each, therefore, as far as can be judged from these experiments, the specific heat of superheated steam is independent of pressure.

Table III gives the relation between specific heat and temperature as derived from the curves in fig. 5, and fig. 6 shows this relation graphically, ordinates representing specific heat, and abscissæ representing temperatures.

Fig. 5.

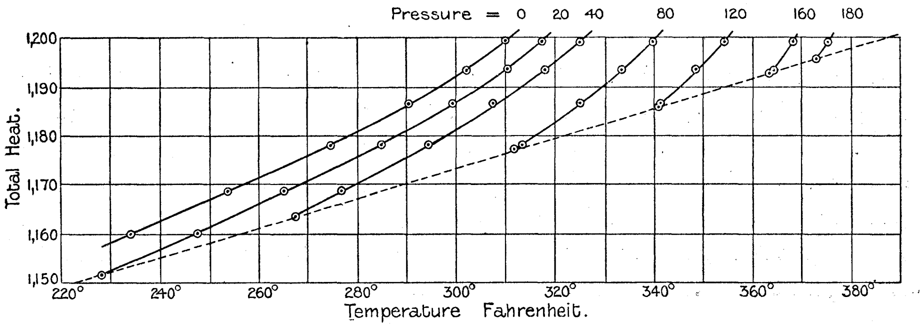


Table II.

Pressure = 0.		Pressure = 20.		Pressure = 40.		Pressure = 80.	
H.	*t.*	H.	*t.*	H.	*t.*	H.	*t.*
1199·3	310·0	1199·3	317·8	1199·4	325·1	1199·3	339·8
1193·6	302·3	1193·6	310·4	1193·6	318·0	1193·5	333·3
1186·8	290·5	1186·8	299·0	1186·7	307·4	1186·8	324·6
1178·1	275·0	1178·1	285·0	1178·0	294·2	1177·9	313·4
1168·6	253·8	1168·6	265·0	1168·6	276·6	1177·0	311·8
1160·1	234·1	1160·1	247·5	1163·4	267·1		
		1151·5	227·9				

Pressure = 120.		Pressure = 160.		Pressure = 180.	
H.	*t.*	H.	*t.*	H.	*t.*
1199·1	354·2	1199·0	368·3	1198·9	375·2
1193·4	348·5	1193·3	364·2	1195·7	373·0
1186·5	341·5	1192·8	363·4		
1186·0	341·0				

Fig. 6.

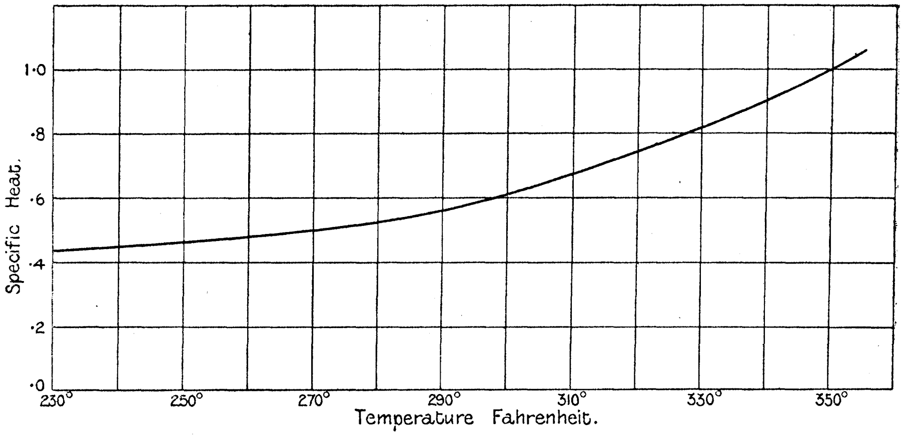


Table III also contains, for comparison's sake, the values of the specific heat given by Grindley, whose results, also, are dependent on Regnault's tables. It will be at once apparent, on comparing the results given in Table III, that they agree fairly well over that part of the range common to both, showing that a considerable error in the cooling curves does not necessarily vitiate, to any great extent, the value of the final deductions with regard to the value of the specific heat.

Table III.

Present experiments.		Grindley's results.	
Temperature.	Specific heat.* (K_p).	Temperatures between which mean specific heat is taken.	Mean specific heat.* (K_p).
230°	0·43	° °	
240	0·44	230·7—246·5	0·4317
260	0·47	246·5—260·8	0·4778
280	0·53	260·8—269·7	0·5152
300	0·61	269·7—295·0	0·5646
320	0·75	295·0—311·5	0·6482
340	0·91		
350	1·00		

The apparent rapid increase of specific heat with temperature to values far beyond any obtained by any other experiments known to the author, led him to suspect the accuracy of Regnault's tables, and caused him to return to the direct heating method as mentioned earlier, with the hope, also, of getting results for higher temperatures than were possible by the throttling experiments.

The existing apparatus was altered as follows, so as to be able to be used in the new experiments. The orifice disc, and most of the gauze above it, were removed; the tube which led to pressure gauge, p_2, and to the thermometer which gave the corresponding saturation temperature, was also taken away, and the broken joint plugged up.

The steam, instead of blowing into the atmosphere, was led through a surface condenser, consisting of one long, straight tube, inside and concentric with a larger one, through which cold water circulated, and then into one or two glass flasks, also immersed in cold, circulating water; these flasks could be removed easily from time to time, and weighed, and by weighing one of the flasks while the other was filling, the experiment could be made quite continuous during a time sufficient to collect several flasks full of condensed steam.

The straight tube surface condenser cooled the steam sufficiently to allow the flask to be open to the atmosphere without losing steam by leakage, and hence the condenser was always at atmospheric pressure.

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* As deduced on the assumption that Regnault's values of the total heat of saturated steam may be accepted as correct.

The electric heater was made of No. 24 German-silver wire, wound in coils on small glass tubes, the whole being enclosed in a glass tube 1 inch in diameter and 8 inches long; this tube was placed in the centre of the separator, with its upper edge placed against the asbestos washer, where the orifice disc was originally, so that all the steam passing through the apparatus was compelled to pass over the heater. Before, however, the steam could reach the heater from the separator, it was compelled to pass along two concentric passages, which enclosed the tube containing the heater; by this means it was sought to avoid loss of heat by radiation, the steam coming in towards the heater taking with it the heat which was being radiated to the walls of the passages.

The electric current was led to the heater by two stout brass conductors, which passed through the cast-iron base of the separator, and were insulated therefrom by conical slate plugs let into the iron.

The room available for the heater was unfortunately very small, and the wires were in consequence crowded rather closely together, with the result that in one or two instances, when very hot, failure of the apparatus was caused through short circuits.

The input of electrical energy was measured by means of an ammeter and voltmeter, which were calibrated from time to time on a Crompton potentiometer, and the necessary corrections applied in the calculations.

Mercury thermometers were used as before, reading in degrees Centigrade, but reading to higher temperatures, the corrections at these higher temperatures were found to be surprisingly large; the calibration of the thermometers was a matter of considerable difficulty, they were, however, compared with the standard platinum resistance thermometer mentioned earlier. The comparison was made by immersing the thermometers in cylinder oil, heated by a gas flame and stirred by means of a small motor. This method of comparing thermometers in a well-stirred fluid appeared to be the only satisfactory one among many attempts.

It was found impossible to entirely eliminate radiation losses, and after a few preliminary experiments, the method of procedure was to aim at keeping steady the temperatures of the steam before and after heating, and then as quickly as possible to obtain the connection between grammes of steam passing per minute and watts required to maintain this constant difference of temperature, this being done for several rates of flow of steam differing over a considerable range. The connection between grammes per minute passing and the input of electrical energy in watts, for a definite temperature rise, was then plotted on squared paper; the points thus obtained would lie on a straight line, which, however, did not pass through the origin, but cut the axis of watts at a height corresponding to the radiation loss expressed in watts. From the slope of the curve, or the increase of watts necessary to maintain a given temperature rise when a definite increase in the flow of steam took place, the specific heat of superheated steam at constant pressure could be calculated by the formula

$$K_p = \frac{\text{Electrical input in watts} \times 0{\cdot}236}{\text{Grammes of steam passing per second} \times \text{temp. rise } {}^\circ\text{C}}.$$

In practice it was found impossible to so adjust the watts that the temperature rise was constant throughout any one series of experiments on account of the time necessary; the watts were rapidly adjusted however to produce a rise as near as possible to the one aimed at, and then the watts were corrected for this particular temperature rise on the assumption that for such slight differences in temperature the rise was directly proportional to the watts. Even then a considerable time was required for all the conditions to become steady before every point obtained.

The difficulty of keeping all the conditions constant during the long time necessary for a complete set of points was always considerable, many of the experiments being spoilt through failure of one condition.

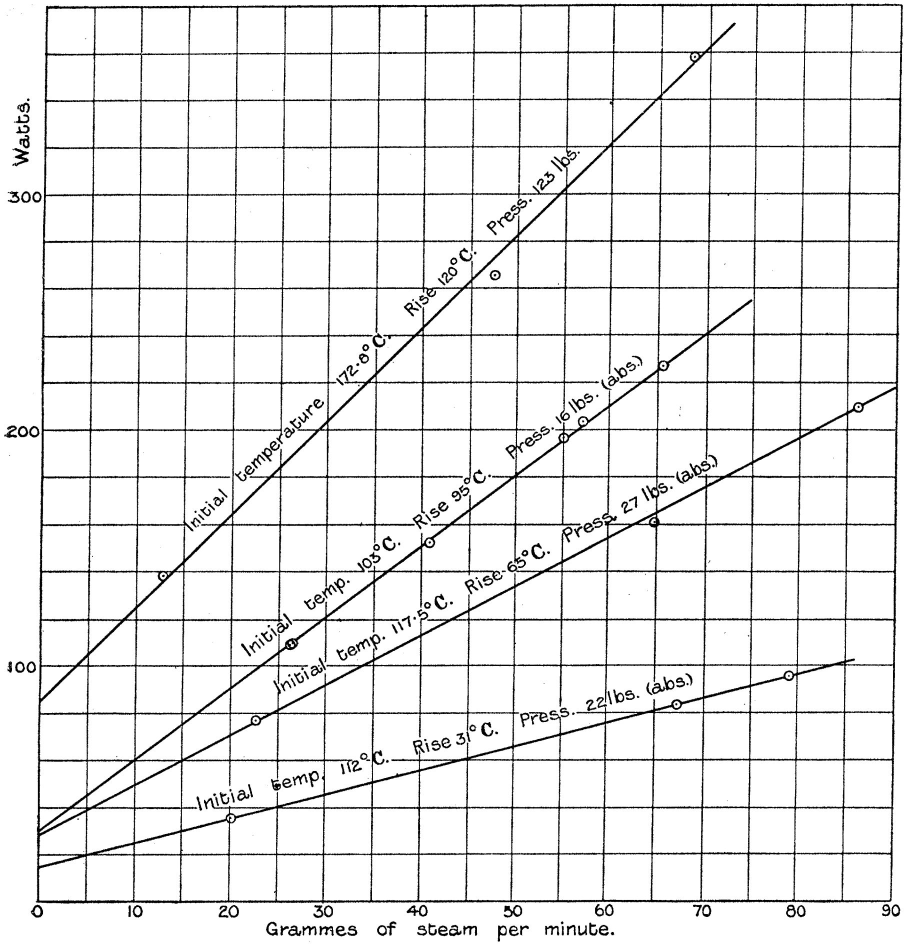
Table IV gives the data obtained in a few characteristic experiments, and

Table IV.

Mean initial temperature.	Mean final temperature.	Rise in °C.	Watts.	Watts for rise of—	Grammes of steam per min.	K_p.
				(31°)		
112°·8	144°·2	31·4	34·8	34·4	20·1	0·473
111·6	142·4	30·8	83·1	83·7	67·2	
111·7	142·6	30·9	95·1	95·4	79·1	
				(65°)		
117·5	182·6	65·1	76·6	76·5	22·9	0·452
117·6	182·7	65·1	160·4	160·2	64·7	
117·4	183·1	65·7	211·5	209·3	86·0	
				(95°)		
102·7	196·3	93·6	106·5	108·1	26·4	0·458
102·7	197·8	95·1	109·5	109·4	26·6	
102·7	198·6	95·9	153·5	152·1	40·9	
102·6	199·5	96·9	200·5	196·5	55·1	
102·7	196·3	93·6	200·5	203·5	57·1	
102·7	200·7	98·0	235·0	227·8	65·4	
				(120°)		
173·0	289·6	116·6	134·3	138·3	13·0	0·459
172·9	293·7	120·8	267·0	265·5	47·3	
172·5	297·0	124·5	372·0	358·2	68·3	

the resulting value of the specific heat thus obtained. The connection between watts and flow for a definite rise in temperature is also shown graphically in fig. 7 for the same experiments.

Fig. 7.



Numerous experiments were carried out, but the results varied too much amongst themselves to allow any conclusions to be drawn as to the manner in which the specific heat may vary with pressure or temperature, except that any such variation must be small, and by no means of the order indicated by the results of the throttling experiments based on Regnault's tables.

The mean of all the results gave $K_p = 0{\cdot}43$, but the author believes that the value 0·46 is more accurate, this being the average of the values obtained in the most satisfactory experiments.

The experiments were all carried out in the Engineering Laboratory of Cambridge University, and the author is indebted to Professor Ewing, F.R.S., for much kindly advice and encouragement given during the course of the research.

On the Distribution of Velocity in a Viscous Fluid over the Cross-Section of a Pipe, and on the Action at the Critical Velocity.

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(Communicated by Professor H. S. Hele-Shaw, F.R.S. Received March 3,—Read March 30, 1905.)

1. *Introduction.*—An expression for the velocity at which stream-line motion breaks down in cylindrical pipes has been obtained by Osborne Reynolds,* and together with many others he has measured the fall of pressure occurring in different lengths. Lord Kelvin,† Lord Rayleigh,‡ and Reynolds§ have investigated the stability of different kinds of flow for viscous and non-viscous fluids, but how nearly the theoretical conditions of velocity distribution actually occur has not hitherto formed the subject of research. Experiments have often been made on the variation of mean linear velocities in the eddying state, but when the motion is irrotational, these are too low to admit of accurate measurement by any method so far employed.

The objects of the present paper are to determine these velocities, to investigate the change which takes place at the critical velocity, and to find the relation between the velocity and the pressure to which it gives rise in a "Pitot" gauge of the form used.

The research has become possible chiefly through the aid of an exceedingly delicate pressure gauge recently designed by my colleagues Professor A. P. Chattock and Mr. J. D. Fry. I am indebted to both these gentlemen for much valuable advice.

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* 'Phil. Trans.,' 1883, Part III, p. 935.

† 'Phil. Mag.,' August, 1887.

‡ 'Phil. Mag.,' July, 1892, etc.

§ 'Phil. Trans.,' A, 1895, vol. 186, p. 123.